Maintaining a reputation through costly signaling

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Overview

In a bilateral relationship with monitoring, a willingness to send costly signals to maintain the relationship reflects a high value of future cooperation, which in turn reflects positively on the sender.

- Types that "behave well" expect to maintain cooperation for longer, leading to higher future value.
- This project: signaling a high continuation value in order to improve one's reputation.

Key ingredients:

- Possibility that cooperating becomes permanently more costly
- Imperfect monitoring of action/action set (strategically equivalent)
- Ability to generate a costly signal with no benefit

Overview

Some motivating examples:

- Client/agency relationship, e.g. a consulting firm maintaining ties with a company it provides services to.
- Buyer/seller with advertising as a signal

Main takeaway is that signaling can allow highly cooperative types to separate from low-cooperation types in an efficiency-improving way.

- Set of outcomes depends on how costly the signaling channel is.
  - Dependence may be finicky
- Without signals, cooperation may break down while it is still profitable. Signals make better equilibria possible.

Setup

2 players, P1 (long-lived) and P2 (short-lived).

- Each period's P2 may "hire" P1 to potentially take an action $a_t \in \{0, 1\}$ at cost w to P2 and benefit g to P1.
- P1 has type $\theta \in \{H, L\}$ – "high-cooperation" and "low-cooperation" types with different costs of acting:
  - $c_H \leq 0$, so type H is a commitment type who always acts.
  - On the other hand, $c_L > 0$.
- Outcome $y_t = \begin{cases} b & \text{w. Prob. } pa_t \\ 0 & \text{otherwise.} \end{cases}$
  - Equivalently, with probability p, P1 can do a favor ($a_t = 1$, with cost c_θ / p), which always results in $y_t = b$.

Type evolution

Every period, there is probability q that a type-H P1 transitions to type L.

Underlying assumption: there is a pool of P1s, small proportion of type H.

- Type H = access to high-quality inputs, high intrinsic motivation

Conditions for type H are impermanent – can worsen due to a negative shock. Opposite could also occur, but shocks are asymmetric:

- Probability of transitioning to type H as an L-type is much smaller than vice-versa (easier to lose an edge than gain it).
- Prospect of transitioning to H has negligible impact on expected payoffs.

Formation of a relationship

Costly for P1 to enter a relationship with P2: fixed cost F. Prior about P1's type is formed conditional on observing entry.

- Let $V^{\theta}(\pi_0, \sigma)$ be the value to type θ of entering into the relationship under equilibrium σ when π_0 is the prior about their type.
- When $F \in [V^L(1, \sigma), V^H(1, \sigma)]$, then the entry equilibrium involves only type H paying F to enter.

But, could assume otherwise; then L can sometimes enter, resulting in a prior $\pi_0 < 1$ at the start.

- L always mixes, so $V^L(\pi_0, \sigma) = F$.

Signaling

Add a costly signaling technology:

- Prior to P2's decision to hire, P1 can send a signal through a channel of fixed cost k (e.g., advertising, schmoozing...).
- No direct benefit.
- Both commitment type H and high-cost type L may choose to signal or not in each period.
  - i.e. H strategizes in signaling, but not in choosing their action.

Timing

1. P1 chooses whether to send a signal, $s_t \in \{0, 1\}$, to P2 at cost k.
2. P1 observes whether Nature chose to switch their type to L last period (if they were type H at the time).
3. P2 observes if a signal was sent, and chooses to hire P1 or not, at cost w and benefit g to P1. If they fail to hire, the period ends; if they do hire, the following occurs:
    1. P1 is able to take an action $a_t \in \{0, 1\}$ at cost c_θ.
    2. With probability pa_t, the action succeeds and a benefit b is generated for P2, which P2 observes. They do not observe a_t itself.
4. If P1's type is H, with probability q Nature switches it to L (if so, it remains L forever).

Related literature

- **Reputation with imperfect monitoring:** Fudenberg and Levine (1992), Board and Meyer-ter-Vehn (2013)
- **Semi-persistent types:** Horner, Takahashi, and Vielle (2015), Peski and Toikka (2017)
  - These papers study very patient players and don't model costly signals.
  - I differ by adding a signaling technology under a specific semi-persistent type process, while agnostic to δ.
- **Costly signaling in reputation-building:** Kaya (2009), Kartal (2018)
  - These authors discuss building a reputation with costly signaling under fixed types.

Equilibrium and assumptions

I look for Markov-Perfect equilibria, where the state is P2's belief, π.

- Since $P2$s are short-lived, this is without loss for them
- Rules out P1 conditioning on own behavior in ways not directly relevant to P2's decision.
- Equilibrium is $\sigma = (s^H(\pi), s^L(\pi), x(\pi), a^H(\pi), a^L(\pi))$.

Payoff assumptions:

- Cooperation with type H is positive-surplus for P2: $pb \geq w$
- c_L is large enough that there is a unique no-signaling equilibrium in which the low type never takes an action.[1]

[1] A sufficient condition for uniqueness is that $c_L \geq \frac{a \log\left(\frac{w}{2pb}\right)(2pb-w)}{w \log(1-q)}$.

Beliefs: action stage

Denote by π_t P2's belief at the start of period t, and by $\tilde{\pi}_t$ their belief following the signaling stage and P1 being informed of any changes to their type. Given that $a^H(\pi) = 1$ and $a^L(\pi) = 0$ for all π, we may relate $\tilde{\pi}_t$ to next-period beliefs π_{t+1} as follows:

Clearly,

$$\pi_{t+1}(\tilde{\pi}_t, x_t = 1, y_t = b) = 1.$$

Also,

$$\pi_{t+1}(\tilde{\pi}_t, x_t = 0, \cdot) = \tilde{\pi}_t,$$

$$\pi_{t+1}(\tilde{\pi}_t, x_t = 1, y_t = 0) = \frac{(1-p)\tilde{\pi}_t}{(1-p)\tilde{\pi}_t + (1-\tilde{\pi}_t)}.$$

Beliefs: signaling stage

The probability of signaling as type H or L, respectively, are $s^H(\pi)$ and $s^L(\pi)$. Then, at the hiring stage, P2 updates their beliefs relative to the start of the period as follows:

$$\tilde{\pi}_t(\pi_t, s_t = 1) = (1-q)\frac{s^H(\pi)\pi}{s^H(\pi)\pi + s^L(\pi)(1-\pi)}$$

$$\tilde{\pi}_t(\pi_t, s_t = 0) = (1-q)\frac{(1-s^H(\pi))\pi}{(1-s^H(\pi))\pi + (1-s^L(\pi))(1-\pi)}$$

Suppressing σ, denote by $V^\theta(\pi)$ and $\tilde{V}^\theta(\tilde{\pi})$ the expected future value of the relationship to P1 at the beginning of the period and at the hiring stage, respectively.

Play in the signaling stage

5 cases for play at the signaling stage:

1. **Neither H nor L signal:** $V^\theta(\pi) = \tilde{V}^\theta((1-q)\pi)$.
2. **Type H always signals, type L never does**:
   $V^H(\pi) = \tilde{V}^H(1-q) - k$ and $V^L(\pi) = \tilde{V}^L(0) = 0$.
3. **Type H always signals, type L sometimes does**: Exists y s.t.
   $V^H(\pi) = \tilde{V}^H(y) - k$ and $V^L(\pi) = \tilde{V}^L(y) - k = \tilde{V}^L(0) = 0$.
4. **Type H sometimes signals, type L does not**: Exists z s.t.
   $V^H(\pi) = \tilde{V}^H(z) = \tilde{V}^H(1-q) - k$ and $V^L(\pi) = \tilde{V}^L(z)$.
5. **Both types always signal**: $V^\theta(\pi) = \tilde{V}^\theta((1-q)\pi) - k$.

P2's hiring problem

P2's decision is simple: their value of hiring P1 is $\tilde{\pi}_t pb - w$, so

$$w(\tilde{\pi}) = \begin{cases} 1 & \tilde{\pi} > \frac{w}{pb} \\ \alpha \in [0,1] & \tilde{\pi} = \frac{w}{pb} \\ 0 & \tilde{\pi} < \frac{w}{pb}. \end{cases}$$

No-signaling benchmark

Suppose we shut down signaling. Then always $\tilde{\pi}_t = (1-q)\pi_t$. The equilibrium σ^{ns} is characterized by:

- Belief is a function of # periods since last success (n):
  - $\pi(n) = \frac{(1-q)^n(1-p)^{n-1}}{(1-q)^{n-1}(1-p)^{n-1}+q\sum_{i=0}^{n-2}(1-q)^i(1-p)^i}$.
  - There is a $N = \min\{n : \pi(n) < \frac{w}{pb}\}$ such that P2 stops hiring iff it has been at least N periods since the last observed success.
- In the long-run, cooperation always breaks down
- Positive probability of premature breakdown, i.e. breakdown while P1 is still type H.

Preventing breakdowns

Can prevent cooperation from breaking down while P1 is still type H if we can separate L from H at some π at which

2. **Type H always signals, type L never does**: $V^H(\pi) = \tilde{V}^H(1) - k$ and $V^L(\pi) = \tilde{V}^L(0) = 0$.

Claim

Let σ_n^{sep} be a strategy profile in which P1 never signals for any $\pi > \pi(n)$, and only type H signals at $\pi(n)$ (and P2 best responds).
The range of costs k such that $g\frac{1-\delta^{n+1}}{1-\delta} \leq k \leq V^H(1-q, \sigma_n^{sep})$ is nonempty, and given any such k, σ_n^{sep} is an equilibrium.

Efficiency comparison

Some signaling equilibria (weakly) improve outcomes for player of each type at each time period, and are strictly better for some player/type:

Claim

A signaling equilibrium σ interim Pareto dominates the no-signaling equilibrium σ^{ns} if and only if for all $n < N$ neither type of P1 signals at belief $\pi(n)$, and at $\pi(N)$ the high type sometimes signals.

Efficiency comparison

Sketch of proof (if): Compare outcomes on like paths.

- For each draw of Nature, there is τ such that cooperation would stop at time τ under the no-signaling equilibrium
  - Until τ, play in each period is identical between equilibria.
- If P1 is type H at τ, their continuation value is weakly positive under signaling equilibrium vs. 0 under no-signaling equilibrium; if type is L at τ, then it is 0 under both.
- P2s living after some histories after τ have positive expected utility under signaling equilibrium, due to the possibility of facing a P1 who has redeemed themselves. Under no signaling, their value is 0.

Efficiency comparison

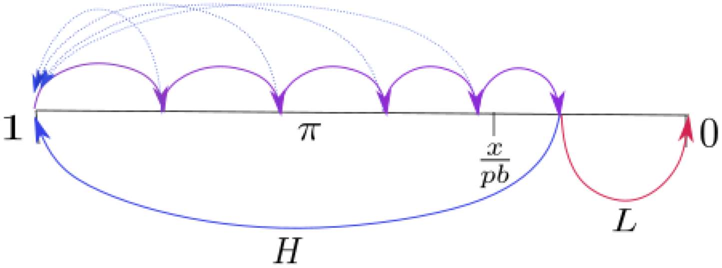
Sketch of proof (only if):

Let σ be a signaling equilibrium and $\hat{n} = \min\{n : s^H(\pi(n)) > 0\}$ be the smallest number of failures in a row such that some type of P1 signals at $\pi(\hat{n})$. Suppose that $\hat{n} < N$. One of 3 cases happens:

- Both types always signal at $\pi(\hat{n})$ – then beliefs update as if neither type signals, with additional cost k at $\pi(n)$.
- $s^H(\pi(\hat{n})) = 1$ and $s^L(\pi(\hat{n})) < 1$ – then L must be indifferent between signaling and quitting, so $V^L(\pi(\hat{n}), \sigma) = 0$; but $V^L(\pi(\hat{n}), \sigma^{ns}) \geq g$.
- $s^H(\pi(\hat{n})) \in (0, 1)$ and $s^L(\pi(\hat{n})) = 0$ – then with positive probability, $\tilde{\pi}(\hat{n}|\sigma) < \tilde{\pi}(\hat{n}|\sigma^{ns})$ and if so, P2 is worse off.

Comparing k: Large k

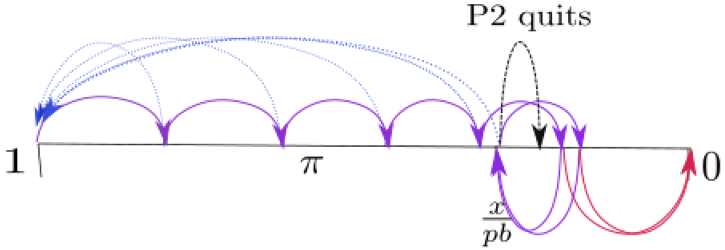
- When $g\frac{1-\delta^{N+1}}{1-\delta} \leq k \leq V^H(1-q, \sigma_N^{sep})$, then σ_N^{sep} is an equilibrium that also Pareto dominates no signaling.



Comparing k: Small k

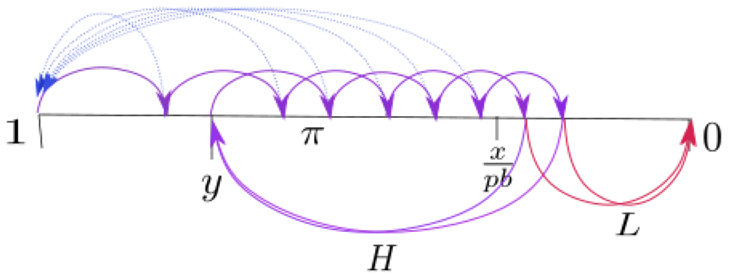
Now consider $k < g$. Excluding profiles in which both types wastefully signal, the unique signaling equilibrium is also Pareto-improving:

- When $\pi < \frac{w}{pb}$, type H always signals, and type L sometimes does, so that $\tilde{\pi} = \frac{w}{pb}$. Upon seeing a signal, P2 hires P1 with probability $\frac{k}{g}$.
- Unique signaling equilibrium because L strictly prefers to signal if doing so results in even 1 extra period of trust from P2.



Other outcomes?

3. **Type H always signals, type L sometimes does**: Exists y s.t. $V^H(\pi) = \tilde{V}^H(y) - k$ and $V^L(\pi) = \tilde{V}^L(y) - k = \tilde{V}^L(0) = 0$.



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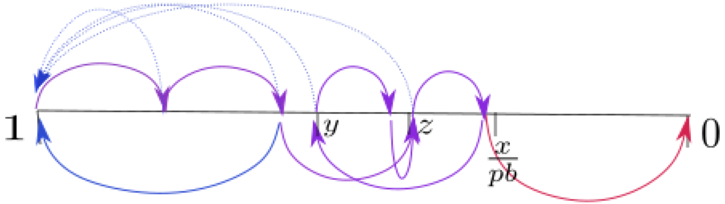


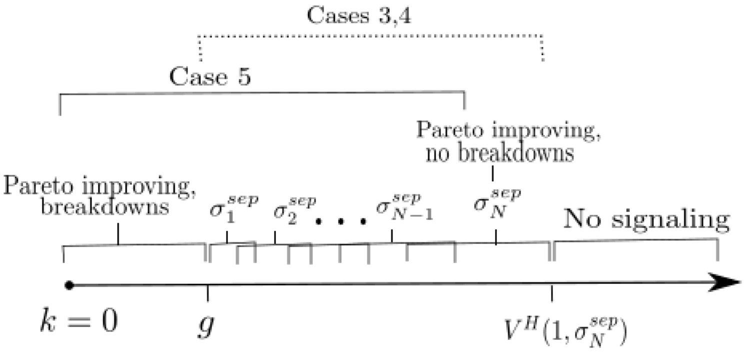
Countability of cases 3, 4

When $k > g$ and case 3 or 4 (either $s^H(\pi) \in (0,1)$ or $s^L(\pi) \in (0,1)$) happen on path, value of k is pinned down by the indifference condition and $\tilde{V}^H(y)$, $\tilde{V}^L(z)$.

- Values $\tilde{V}^H(y)$, $\tilde{V}^L(z)$ themselves depend on the sequence of cases of play in the signaling stages of future periods.
- But, only 5 cases $\Rightarrow$ countable set of possible equations describing $\tilde{V}^H(y)$, $\tilde{V}^L(z)$.
- Therefore, countable number of $k > g$ supporting signaling equilibria with case 3 or 4 on path.

Would not expect these cases to occur "in the wild" if Nature determines k from a continuum/with noise.

Equilibria as a function of k



Relationship to advertising?

Advertising "experience qualities" may be a positive signal of a product's value (Nelson '74, Schmalensee '78, Kihlstrom and Riordan '84).

- Could take this model to be a rational basis for consumers who respond positively to soft advertising.
    - P2 observes a public history of consumer experiences with product line.
    - Imperfect monitoring: element of chance in whether products meet consumer expectations.
    - Firms differ in whether they have an advantage in producing high-quality or low-quality products, and there is drift over time (management shifts, input prices, etc).
- Differs from cited models in focusing on a bilateral relationship and endogenizing consumer behavior (but ignores pricing, competition).

Directions to go

Plan to compare best possible ex-ante welfare under different signal levels.

- Could also consider whether it is optimal to change the cost of the anticipated signal as beliefs vary.

Could be an interesting model with 2 symmetric long-run players who mutually monitor and signal to each other.

- Models an equal partnership rather than a buyer/seller or client/agency relationship

Other thoughts:

- Would like to relax the discreteness of predictions, maybe with smoother types/signals.
- How would signaling would interact with choice to cooperate?